This print-out should have 24 questions. Multiple-choice questions may continue on the next column or page – find all choices before answering. The due time is Central time.

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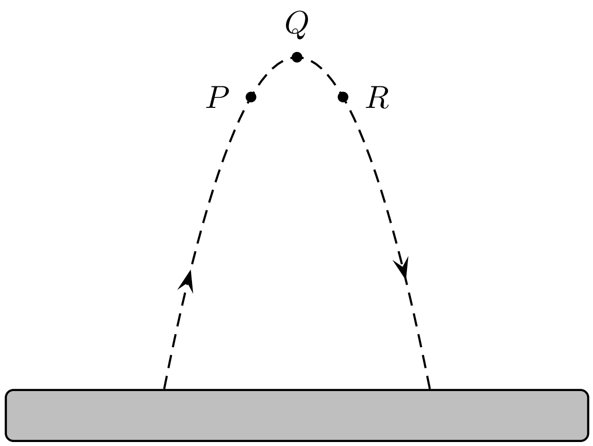
**001** (part 1 of 3) 3 points

A ball is thrown and follows the parabolic path shown. Air friction is negligible. Point $Q$ is the highest point on the path. Points $P$ and $R$ are the same height above the ground.




How do the speeds of the ball at the three points compare?

**1.** $\|\vec{v}_P\| < \|\vec{v}_Q\| < \|\vec{v}_R\|$

**2.** $\|\vec{v}_R\| < \|\vec{v}_Q\| < \|\vec{v}_P\|$

**3.** $\|\vec{v}_Q\| < \|\vec{v}_P\| = \|\vec{v}_R\|$ **correct**

**4.** $\|\vec{v}_P\| = \|\vec{v}_R\| < \|\vec{v}_Q\|$

**5.** $\|\vec{v}_Q\| < \|\vec{v}_R\| < \|\vec{v}_P\|$

**Explanation:**
The speed of the ball in the $x$-direction is constant. Because of gravitational acceleration, the speed in the $y$-direction goes to zero at point $Q$. Since points $P$ and $R$ are located at the same point above ground, by symmetry we see that they have the same speed in the $y$-direction (though they do not have the same velocity). The answer is then "$v_Q < v_P = v_R$".

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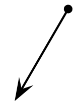
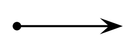
**002** (part 2 of 3) 3 points
Which of the following diagrams best indicates the direction of the acceleration, if any, on the ball at point $R$?

**1.** 

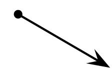
**2.** 

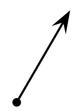
**correct**

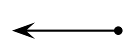
**3.** 

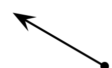
**4.** 

**5.** The ball is in free-fall and there is no acceleration at any point on its path.

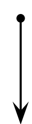
**6.** 

**7.** 

**8.** 

**9.** 

**Explanation:**
Since air friction is negligible, the only acceleration on the ball after being thrown is that due to gravity. The answer is



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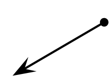
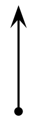
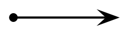
**003** (part 3 of 3) 4 points
Which of the following diagrams best indicates the direction of the net force, if any, on the ball at point $Q$?

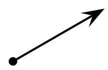
**1.** 

**2.** 

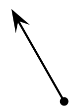
**correct**

**3.** 

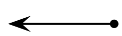
**4.** 

**5.** 

**6.** The ball is in free-fall and there is no acceleration at any point on its path.

**7.** 

**8.** 

**9.** 

**Explanation:**

By Newton's second law, the force must be in the direction of the acceleration. The answer is

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**004** (part 1 of 3) 3 points

A particle starts from the origin at $t = 0$ with an initial velocity having an $x$ component of 29.9 m/s and a $y$ component of $-18$ m/s. The particle moves in the $xy$ plane with an $x$ component of acceleration only, given by 4.6 m/s$^2$.

Determine the $x$ component of velocity after 3.47 s.

Correct answer: 45.862 m/s.

**Explanation:**

With $v_{x0} = 29.9$ m/s and $a_x = 4.6$ m/s$^2$, the equations for the $x$ component of velocity at 3.47 s is

$$\begin{aligned} v_x &= v_{x0} + a_x\, t \\ &= 29.9 \text{ m/s} + (4.6 \text{ m/s}^2)\,(3.47 \text{ s}) \\ &= 45.862 \text{ m/s} . \end{aligned}$$

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**005** (part 2 of 3) 3 points

Find the speed of the particle after 3.47 s.

Correct answer: 49.2679 m/s.

**Explanation:**

The $y$-component of the acceleration of the particle is zero, so $v_y$ is constant

$$v_y = v_{y0} = -18 \text{ m/s} .$$

Therefore the speed $v = \|\vec{v}\|$ of the particle at 3.47 s is

$$\begin{aligned} v &= \sqrt{v_x^2 + v_y^2} \\ &= \sqrt{(45.862 \text{ m/s})^2 + (-18 \text{ m/s})^2} \\ &= 49.2679 \text{ m/s} . \end{aligned}$$

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**006** (part 3 of 3) 4 points

Find the magnitude of the displacement vector of the particle after $t = 3.47$ s.

Correct answer: 145.532 m.

**Explanation:**

Since at $t = 0$, $x_0 = y_0 = 0$, the vector equation

$$\vec{r}(t) = \vec{r}_0 + \vec{v}_0\, t + \frac{1}{2}\,\vec{a}\, t^2$$

gives for $t = 3.47$ s.

$$\begin{aligned} x &= v_{x0}\, t + \frac{1}{2}\, a_x\, t^2 \\ &= (29.9 \text{ m/s})\,(3.47 \text{ s}) \\ &\qquad + \frac{1}{2}\,(4.6 \text{ m/s}^2)\,(3.47 \text{ s})^2 \\ &= 131.447 \text{ m}, \\ y &= v_{y0}\, t \\ &= (-18 \text{ m/s})\,(3.47 \text{ s}) \\ &= -62.46 \text{ m}. \end{aligned}$$

For the magnitude of the displacement vector of the particle at $t = 3.47$ s we have

$$\begin{aligned} r &= \sqrt{x^2 + y^2} \\ &= \sqrt{(131.447 \text{ m})^2 + (-62.46 \text{ m})^2} \\ &= 145.532 \text{ m} . \end{aligned}$$

*Note:* This is *not* the distance that the car travels in this time!

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**007** (part 1 of 4) 3 points

A plane is flying horizontally with speed 263 m/s at a height 6910 m above the ground, when a package is dropped from the plane.

The acceleration of gravity is $9.8 \text{ m/s}^2$.

Neglecting air resistance, when the package hits the ground, the plane will be

**1.** directly above the package. **correct**

**2.** behind the package.

**3.** ahead of the package.

**Explanation:**

Both have the same velocity at the time of release. Gravitational acceleration does not change horizontal velocity, so the plane will be directly above the package.

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**008** (part 2 of 4) 3 points

What is the horizontal distance from the release point to the impact point?

Correct answer: 9876.36 m.

**Explanation:**

For the vertical fall, $h = \dfrac{1}{2}\, g\, t^2$, or $t = \sqrt{\dfrac{2\,h}{g}}$. The horizontal distance traveled is

$$\begin{aligned} x &= v\,t \\ &= v\sqrt{\frac{2\,h}{g}} \\ &= (263 \text{ m/s})\sqrt{\frac{2\,(6910 \text{ m})}{9.8 \text{ m/s}^2}} \\ &= 9876.36 \text{ m}\,. \end{aligned}$$

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**009** (part 3 of 4) 3 points

A second package is thrown downward from the plane with a vertical speed $v_1 = 52$ m/s.

What is the magnitude of the total velocity of the package at the moment it is thrown as seen by an observer on the ground?

Correct answer: 268.091 m/s.

**Explanation:**

The velocity is the vector sum of the vertical and horizontal components of velocity as seen from the ground. Hence the scalar speed is

$$\begin{aligned} s &= \sqrt{v^2 + v_1^2} \\ &= \sqrt{(263 \text{ m/s})^2 + 52 \text{ m/s}^2} \\ &= 268.091 \text{ m/s}\,. \end{aligned}$$

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**010** (part 4 of 4) 3 points

What horizontal distance is traveled by this package?

Correct answer: 8578.95 m.

**Explanation:**

The time of the vertical fall is now determined by

$$\begin{aligned} h &= v_1\,t + \frac{1}{2}\,g\,t^2 \\ 0 &= \frac{1}{2}\,g\,t^2 + v_1\,t - h \\ t &= \frac{-v_1 + \sqrt{v_1^2 + 4\,(\frac{1}{2})\,g\,h}}{2\,\frac{1}{2}\,g} \\ &= \frac{-v_1 + \sqrt{v_1^2 + 2\,g\,h}}{g} \\ &= 32.6196 \text{ s}\,. \end{aligned}$$

The horizontal distance is

$$\begin{aligned} x &= v\,t \\ &= (263 \text{ m/s})\,(32.6196 \text{ s}) \\ &= 8578.95 \text{ m}\,. \end{aligned}$$

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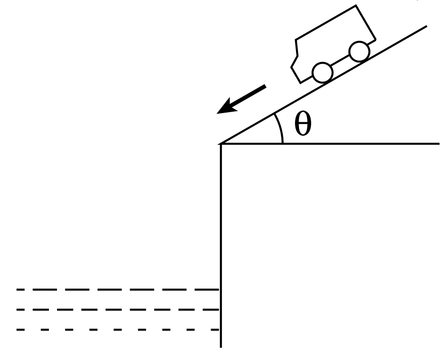
**011** (part 1 of 1) 10 points

A car is parked near a cliff overlooking the ocean on an incline that makes an angle of 33° with the horizontal. The negligent driver leaves the car in neutral, and the emergency brakes are defective. The car rolls from rest down the incline and has a velocity 6 m/s when it reaches the edge of the cliff. The cliff is 47.4 m above the ocean.

The acceleration of gravity is $9.8 \text{ m/s}^2$.



How far is the car from the base of the cliff when the car hits the ocean?
Correct answer: 14.0624 m.
**Explanation:**
First, find the car's initial vertical velocity when it leaves the cliff

$$\begin{aligned} v_{0y} &= v \sin\theta \\ &= (6 \text{ m/s}) \sin(33^\circ) \\ &= 3.26784 \text{ m/s}. \end{aligned}$$

Then find the vertical velocity with which the car strikes the water as

$$\begin{aligned} v_y^2 &= v_{0y}^2 + 2\,g\,h \\ &= (3.26784 \text{ m/s})^2 + 2\,(9.8 \text{ m/s}^2)\,(47.4 \text{ m}) \\ &= 939.719 \text{ m}^2/\text{s}^2, \end{aligned}$$

or $v_y = 30.6548$ m/s. The time of flight is found from

$$v_y = v_{0y} + g\,t$$

as

$$t = \frac{v_y - v_{0y}}{g} = 2.79459 \text{ s}.$$

The initial velocity is $v_{0x} = v\cos\theta$ and the horizontal motion of the car during this time is

$$\begin{aligned} x &= v_{0x}\,t \\ &= (6 \text{ m/s}) \cos(33^\circ)\,(2.79459 \text{ s}) \\ &= 14.0624 \text{ m}. \end{aligned}$$

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**012** (part 1 of 1) 10 points
A brick is thrown upward from the top of a building at an angle of 39.5° above the horizontal and with an initial speed of 7.04 m/s.
The acceleration of gravity is 9.8 m/s$^2$.
If the brick is in flight for 3.2 s, how tall is the building?
Correct answer: 35.8464 m.
**Explanation:**
**Basic Concept**
The height of the building is determined by the vertical motion with gravity acting down and an initial velocity acting upward:

$$y = y_0 + v_{0y}\,t - \frac{1}{2}\,g\,t^2$$

**Solution**
Choose the origin at the base of the building. The initial position of the brick is $y_0 = h$, the vertical component of the initial velocity is $v_{0y} = v_0 \sin\theta$ directed upward, and $y = 0$ when the brick reaches the ground, so

$$0 = h + v_{0y}\,t - \frac{1}{2}\,g\,t^2$$

$$\begin{aligned} h &= -v_{0y}\,t + \frac{1}{2}\,g\,t^2 \\ &= -(4.47799 \text{ m/s})\,(3.2 \text{ s}) \\ &\quad + \frac{1}{2}\,\left(9.8 \text{ m/s}^2\right)\,(3.2 \text{ s})^2 \\ &= 35.8464 \text{ m}. \end{aligned}$$

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**013** (part 1 of 1) 10 points
A pitched ball is hit by a batter at a 46° angle. It just clears the outfield fence, 99 m away.
The acceleration of gravity is 9.8 m/s$^2$.
Find the velocity of the ball when it left the bat. Assume the fence is the same height as the pitch.
Correct answer: 31.1575 m/s.
**Explanation:**
**Basic Concepts**
Horizontally,

$$v_{oh} = v\cos\theta$$

$$v_h = v_{oh}$$

$$d = v_{oh} t$$

Vertically,

$$v_{ov} = v\sin\theta$$

$$v_v = v_{ov} - gt$$

$$h = v_{ov} t - \frac{1}{2} g t^2$$

**Solution**
At the maximum range of the ball, $v_{fv} = -v_{ov}$, so

$$-v_{ov} = v_{ov} - gt$$

$$-2v_{ov} = -gt$$

$$t = 2\frac{v_{ov}}{g}$$

The maximum distance covered is

$$d = v_{oh} t = \frac{2 v_{oh} v_{ov}}{g}$$

$$d = \frac{2 \cdot v \cos\theta \cdot v \sin\theta}{g}$$

$$d = \frac{v^2 (2 \sin\theta \cos\theta)}{g} = \frac{v^2 \sin(2\theta)}{g}$$

Thus the initial velocity is

$$v = \sqrt{\frac{dg}{\sin(2\theta)}}$$

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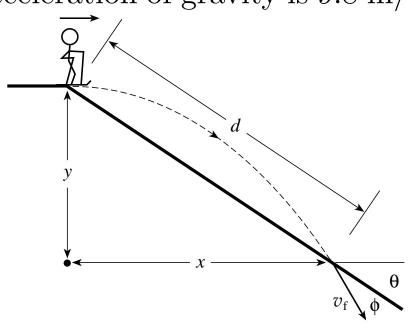
**014** (part 1 of 3) 3 points

A ski jumper travels down a slope and leaves the ski track moving in the horizontal direction with a speed of 26 m/s as in the figure. The landing incline below her falls off with a slope of $\theta = 56.9^\circ$ .

The acceleration of gravity is $9.8 \text{ m/s}^2$ .




Calculate the distance $d$ she travels along the incline before landing.

Correct answer: 387.527 m.

**Explanation:**

It is convenient to select the origin ($x = y = 0$) at the beginning of the jump. Since $v_{x0} = 26$ m/s and $v_{y0} = 0$ m/s in this case, we have

$$x = v_{x0}\, t$$
$$y = v_{y0}\, t - \frac{1}{2}\, g\, t^2 = \frac{1}{2}\, g\, t^2 .$$

The distance $d$ she travels along the incline before landing is related to her $x$ and $y$ coordinates by

$$x = d \cos\theta$$
$$y = -d \sin\theta .$$

Substituting these expressions for $x$ and $y$ into the two equations above, we obtain

$$d \cos\theta = v_x\, t$$
$$d \sin\theta = \frac{1}{2}\, g\, t^2 .$$

Excluding $t$ from these equations gives

$$\begin{aligned} d &= \frac{2\, v_0^2 \sin\theta}{g \cos^2\theta} \\ &= \frac{(2)\,(26 \text{ m/s})^2 \sin 56.9^\circ}{(9.8 \text{ m/s}^2) \cos^2 56.9^\circ} \\ &= 387.527 \text{ m} . \end{aligned}$$

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**015** (part 2 of 3) 3 points

Determine how long the ski jumper is airborne.

Correct answer: 8.13958 s.

**Explanation:**

Excluding $d$ rather than $t$ from the system above, we obtain

$$\begin{aligned} t &= \frac{2\, v_0 \tan\theta}{g} \\ &= \frac{(2)\,(26 \text{ m/s}) \tan(56.9^\circ)}{9.8 \text{ m/s}^2} \\ &= 8.13958 \text{ s} . \end{aligned}$$

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**016** (part 3 of 3) 4 points

What is the relative angle $\phi$ with which the ski jumper hits the slope?

Correct answer: 15.0469 °.

**Explanation:**

The direction $\phi_t$ of the velocity vector (relative to the positive $x$ axis) at impact is

$$\begin{aligned} \phi_t &= \arctan\left(\frac{v_y}{v_x}\right) \\ &= \arctan\left(\frac{-79.7679 \text{ m/s}}{26 \text{ m/s}}\right) \\ &= -71.9469^\circ , \end{aligned}$$

where $v_y = -g\, t$ and $v_x = v_0$. Therefore the relative angle of impact $\phi$ on the slope is

$$\begin{aligned} \phi &= |\phi_t| - \theta \\ &= 71.9469^\circ - 56.9^\circ \\ &= 15.0469^\circ . \end{aligned}$$

**017** (part 1 of 1) 10 points

A stone is thrown horizontally at 13 m/s from a cliff 79.2 m high.

The acceleration of gravity is 9.8 m/s$^2$ .

How far from the base of the cliff does the stone strike the ground?

Correct answer: 52.2646 m.

**Explanation:**

**Basic concepts**

For the vertical motion, all of the motion is directed downward, $v_{oy} = 0$ and we can let $y_o = 0$.

For the horizontal motion, $a_x = 0$ and we can let $x_o = 0$.

The time of flight is the only value in common for the vertical and horizontal part of the motion.

**Solution**

For the vertical motion

$$y = \frac{1}{2} g t^2$$

$$2y = g t^2$$

$$t^2 = \frac{2y}{g}$$

$$t = \sqrt{\frac{2y}{g}}$$

The distance covered horizontally during this time is

$$x = v_x t = v \sqrt{\frac{2y}{g}}$$

**018** (part 1 of 1) 10 points

A ball on the end of a string is whirled around in a horizontal circle of radius 0.444 m. The plane of the circle is 1.98 m above the ground. The string breaks and the ball lands 2.06 m away from the point on the ground directly beneath the ball's location when the string breaks.

The acceleration of gravity is 9.8 m/s$^2$ .

Find the centripetal acceleration of the ball during its circular motion.

Correct answer: 23.6528 m/s$^2$.

**Explanation:**

In order to find the centripetal acceleration of the ball, we need to find the initial velocity of the ball. Let $y$ be the distance above the ground. After the string breaks, the ball has no initial velocity in the vertical direction, so the time spent in the air may be deduced from the kinematic equation,

$$y = \frac{1}{2} g t^2$$

Solving for $t$,

$$\Rightarrow t = \sqrt{\frac{2y}{g}}$$

Let $d$ be the distance traveled by the ball. Then

$$v_x = \frac{d}{t} = \frac{d}{\sqrt{\frac{2y}{g}}}$$

Hence, the centripetal acceleration of the ball during its circular motion is

$$a_c = \frac{v_x^2}{r} = \frac{d^2 g}{2 y r} = 23.6528 \text{ m/s}^2$$

**019** (part 1 of 2) 5 points

The orbit of a Moon about its planet is approximately circular, with a mean radius of $4.31 \times 10^8$ m. It takes 41.1 days for the Moon to complete one revolution about the planet.

Find the mean orbital speed of the Moon.

Correct answer: 762.608 m/s.

**Explanation:**

Dividing the length $C = 2\pi r$ of the trajectory of the Moon by the time

$$T = 41.1 \text{ days } = 3.55104 \times 10^6 \text{ s}$$

of one revolution (in seconds!), we obtain that the mean orbital speed of the Moon is

$$\begin{aligned} v &= \frac{C}{T} = \frac{2\,\pi\, r}{T} \\ &= \frac{2\,\pi\,(4.31 \times 10^8 \text{ m})}{3.55104 \times 10^6 \text{ s}} \\ &= 762.608 \text{ m/s} . \end{aligned}$$

**020** (part 2 of 2) 5 points

Find the Moon's centripetal acceleration.
Correct answer: 0.00134935 $\text{m/s}^2$.

**Explanation:**
Since the magnitude of the velocity is constant, the tangential acceleration of the Moon is zero. For the centripetal acceleration we use the formula

$$\begin{aligned} a_c &= \frac{v^2}{r} \\ &= \frac{(762.608 \text{ m/s})^2}{4.31 \times 10^8 \text{ m}} \\ &= 0.00134935 \text{ m/s}^2 . \end{aligned}$$

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**021** (part 1 of 2) 5 points
A river flows at a speed $v_r = 5.97$ km/hr with respect to the shoreline. A boat needs to go perpendicular to the shoreline to reach a pier on the river's other side. To do so, the boat heads upstream at an angle $\theta = 38^\circ$ from the direction to the boat's pier.

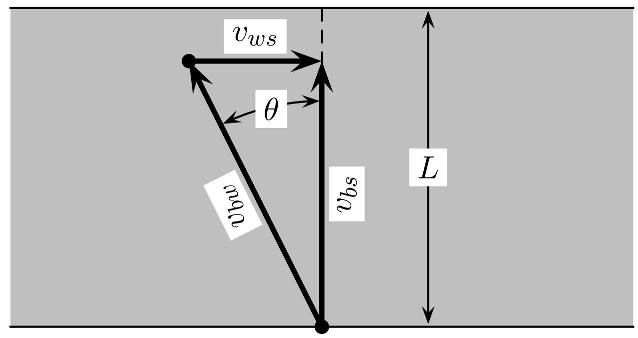
Find the ratio of $v_b$ to $v_r$, where $v_r$ is defined above and $v_b$ is the boat's speed with respect to the water.

1. $\dfrac{v_b}{v_r} = \sin^2\theta$
2. $\dfrac{v_b}{v_r} = \tan\theta$
3. $\dfrac{v_b}{v_r} = \dfrac{1}{\sin^2\theta}$
4. $\dfrac{v_b}{v_r} = \dfrac{1}{\cos^2\theta}$
5. $\dfrac{v_b}{v_r} = \dfrac{1}{\tan\theta}$
6. $\dfrac{v_b}{v_r} = \dfrac{1}{\cos\theta}$
7. $\dfrac{v_b}{v_r} = \cos\theta$
8. $\dfrac{v_b}{v_r} = \cos^2\theta$
9. $\dfrac{v_b}{v_r} = \sin\theta$
10. $\dfrac{v_b}{v_r} = \dfrac{1}{\sin\theta}$ **correct**

**Explanation:**

$$\begin{aligned} \text{Let}: \quad v_{bs} &= ? , \ \text{boat relative to shore} \\ v_{ws} &= v_r , \ \text{water relative to shore} \\ v_{bs} &= v_b , \ \text{boat relative to water} . \end{aligned}$$

This is a problem about relative velocity. See the figure below.




The velocity of the boat relative to the shore is given by

$$\overrightarrow{v_{bs}} = \overrightarrow{v_{bw}} + \overrightarrow{v_{ws}} .$$

It is easy to see from the figure above that $v_{ws}$, $v_{bw}$ and $v_{bs}$ form a right triangle if the boat moves northward relative to the earth. Therefore,

$$v_{ws} = v_{bw}\sin\theta$$
$$\Longrightarrow \frac{v_{bw}}{v_{ws}} = \frac{1}{\sin\theta} .$$

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**022** (part 2 of 2) 5 points
If the time taken for the boat to cross the river is 10.6 min, determine the width of the river.
Correct answer: 1.34996 km.

**Explanation:**
By similar reasoning, we know relative to the shore, the velocity of the boat is

$$v_{bs} = \frac{v_{ws}}{\tan\theta} = 7.64125 \text{ km/hr} .$$

Therefore the time taken for the boat to cross the river is given by

$$\begin{aligned} t &= \frac{L}{v_{bs}} \\ &= \frac{L}{\dfrac{v_{ws}}{\tan\theta}} \\ &= \frac{L\tan\theta}{v_{ws}} . \end{aligned}$$

$$\Longrightarrow L = \frac{v_{ws}\,t}{\tan\theta}$$
$$= \frac{(5.97\text{ km/hr})\,(10.6\text{ min})\,(1\text{hr}/60\text{ min})}{\tan 38^\circ}$$
$$= \frac{(5.97\text{ km/hr})\,(0.176667\text{ hr})}{\tan 38^\circ}$$
$$= 1.34996\text{ km}\,.$$

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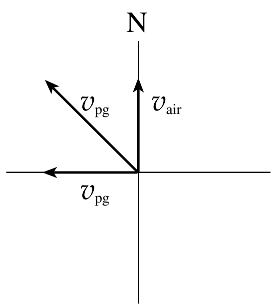
**023** (part 1 of 2) 5 points

An airplane moving in a calm weather (no wind) is headed to the west. If there is a wind of 45.2 km/h toward the north, find the speed of the airplane relative to the ground.

The airplane's speed relative to the air is 168 km/h.

Correct answer: 173.974 km/h.

**Explanation:**




Let $\vec{\mathbf{v}}_{\mathbf{pa}} = -(168\text{ km/h})\,\vec{\mathbf{i}}$ and $\vec{\mathbf{v}}_{\mathbf{air}} = (45.2\text{ km/h})\,\vec{\mathbf{j}}$. The speed of the airplane with respect to ground, ($v_{pg}$), is the magnitude of the vector sum $\vec{\mathbf{v}}_{\mathbf{pa}} + \vec{\mathbf{v}}_{\mathbf{air}}$.

$$v_{pg} = \sqrt{v_{pa}^2 + v_{air}^2} = 173.974\text{ km/h}$$

The direction of $\vec{\mathbf{v}}_{\mathbf{pg}}$ is given by

$$\theta = \tan^{-1}\frac{v_{air}}{v_{pa}} = 15.0587^\circ$$

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**024** (part 2 of 2) 5 points

What is the direction of the speed of the plane with respect to ground? Measure in terms of "north of west".

Correct answer: 15.0587 °.

**Explanation:**